問6 (2)

$\sin 2x = 2\sin x \cos x$ より

$$I_2 = \frac{1}{2}\int e^x \sin 2x \, dx$$

$f' = e^x \qquad f = e^x$

$g = \sin 2x \qquad g' = 2\cos 2x$

$$I_2 = \frac{1}{2}\left(e^x \sin 2x - 2\int e^x \cos 2x \, dx\right)$$

$f' = e^x \qquad f = e^x$

$g = \cos 2x \qquad g' = -2\sin 2x$

$$I_2 = \frac{1}{2}\left(e^x \sin 2x - 2\left(e^x \cos 2x + 2I_2\right)\right)$$

$$2I_2 = e^x \sin 2x - 2e^x \cos 2x - 4I_2$$

$$6I_2 = e^x(\sin 2x - 2\cos 2x)$$

$$I_2 = \frac{e^x}{6}(\sin 2x - 2\cos 2x)$$

(3)

$$I_3 = \int e^{-x} \sin 3x \, dx$$

$f = e^{-x} \qquad f' = -e^{-x}$

$g' = \sin 3x \qquad g = \frac{-1}{3}\cos 3x$

$$I_3 = \frac{-1}{3}e^{-x}\cos 3x - \frac{1}{3}\int e^{-x}\cos 3x \, dx$$

$f = e^{-x} \qquad f' = -e^{-x}$

$g' = \cos 3x \qquad g = \frac{1}{3}\sin 3x$

$$I_3 = -\frac{1}{3}e^{-x}\cos 3x - \frac{1}{3}\left(\frac{e^{-x}}{3}\sin 3x + \frac{1}{3}I_3\right)$$

$$\frac{10}{9}I_3 = -\frac{e^{-x}}{3}\left(\cos 3x + \frac{1}{3}\sin 3x\right)$$

$$I_3 = -\frac{3}{10}e^{-x}\left(\cos 3x + \frac{1}{3}\sin 3x\right)$$

$$\left(= -\frac{e^{-x}}{10}(3\cos 3x + \sin 3x)\right.$$

(4) $I_4 = \int e^x \cos 3x \, dx$

$f = e^x \qquad f' = e^x$

$g' = \cos 3x \qquad g = \frac{1}{3}\sin 3x$

$$I_4 = \frac{1}{3}e^x \sin 3x - \frac{1}{3}\int e^x \sin 3x \, dx$$

$f = e^x \qquad f' = e^x$

$g' = \sin 3x \qquad g = -\frac{1}{3}\cos 3x$

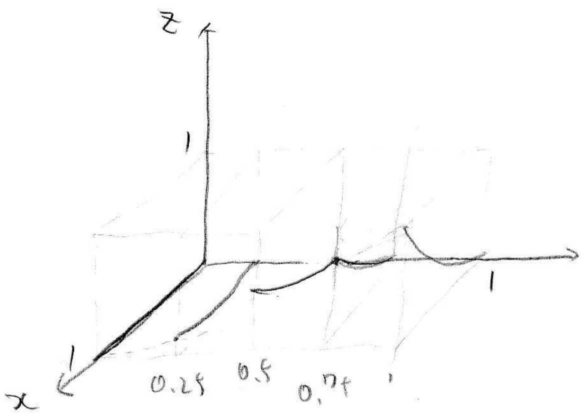
$$I_4 = \frac{1}{3}e^x \sin 3x - \frac{1}{3}\left(-\frac{e^x}{3}\cos 3x + \frac{1}{3}I_4\right)$$

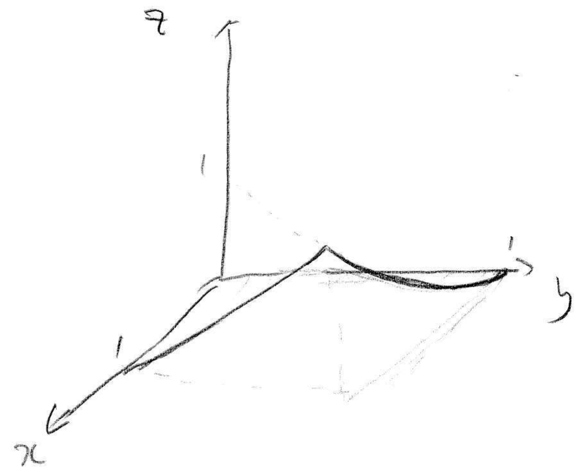
$$\frac{10}{9}I_4 = \frac{1}{3}e^x \sin 3x + \frac{1}{9}e^x \cos 3x$$

$$I_4 = \frac{3}{10}e^x(3\sin 3x + \cos 3x)$$

問5

(1)





(2) $$\int_{y=0}^{y=1}\int_{x=0}^{x=1} x^2 y\,dx\,dy = \int_{y=0}^{y=1}\left[\frac{x^3}{3}y\right]_{x=0}^{x=1} dy$$

$$= \int_{y=0}^{y=1}\frac{y}{3}\,dy = \left[\frac{y}{6}\right]_0^1 = \frac{1}{6}$$

問6

(1) $I_1 = \int e^x \sin x\,dx$

$f = e^x \quad f' = e^x$

$g' = \sin x \quad g = -\cos x$

$I_1 = -e^x\cos x + \int e^x \cos x\,dx$

$f = e^x \quad f' = e^x$

$g' = \cos x \quad g = \sin x$

$I_1 = -e^x\cos x + e^x \sin x - I_1$

$2I_1 = e^x(\sin x - \cos x)$

$I_1 = \frac{e^x}{2}(\sin x - \cos x)$

(1) 別解

$f' = e^x \quad f = e^x$

$g = \sin x \quad g' = \cos x$

$I_1 = e^x \sin x - \int e^x \cos x\,dx$

$f' = e^x \quad f = e^x$

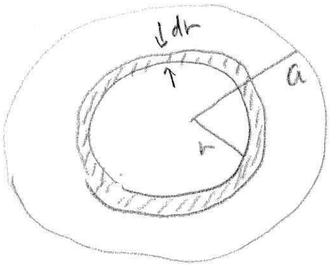
$g = \cos x \quad g' = -\sin x$

$I_1 = e^x \sin x - (e^x\cos x + \int e^x \sin x\,dx)$

$= e^x\sin x - e^x\cos x - I_1$

$\therefore\ I_1 = \frac{1}{2}e^x(\sin x - \cos x)$

問3



半径r,厚さdr の 球殻を考える

球殻の表面積は

$$S = 4\pi r^2$$

球殻の体積は

$$dv = 4\pi r^2 dr$$

球殻の重さは

$$dm = \rho dv = 4\pi r^2 (ar^2 + br + c) dr$$

$$= 4\pi (ar^4 + br^3 + cr^2) dr$$

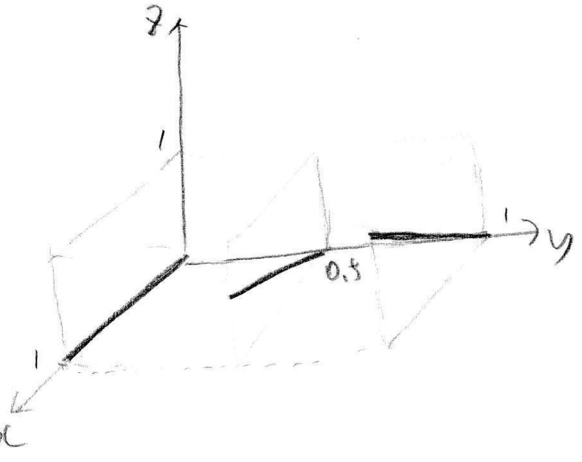
rで積分すると

$$M = \int_{r=0}^{r=R} 4\pi (ar^4 + br^3 + cr^2) dr$$

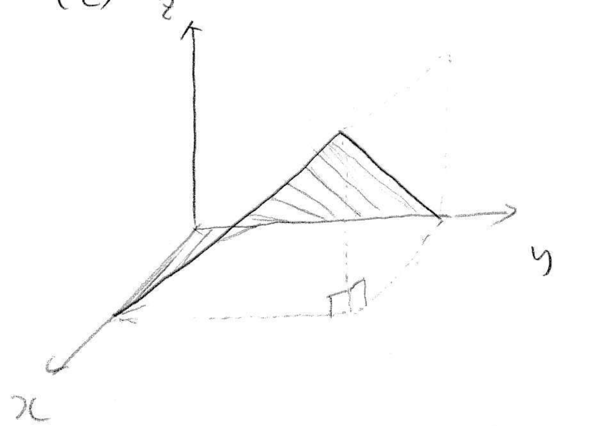
$$= 4\pi \left(\frac{a}{5}R^5 + \frac{b}{4}R^4 + \frac{c}{3}R^3\right)$$

問4

(1)



(2)

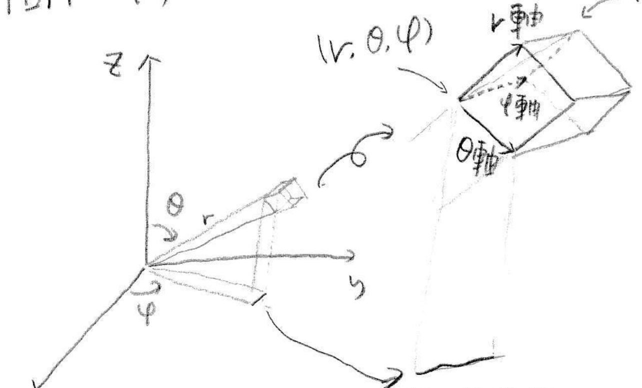


(3) $$\int_{y=0}^{y=1}\int_{x=0}^{x=1} xy\,dx\,dy = \int_{y=0}^{y=1} \left[\frac{x^2}{2}y\right]_{x=0}^{x=1} dy$$

$$= \int_0^1 \frac{y}{2}\,dy = \left[\frac{y^2}{4}\right]_0^1 = \frac{1}{4}$$

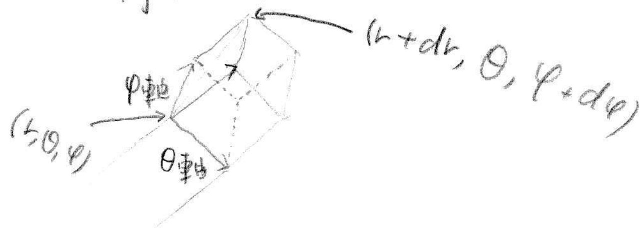
基礎物理I 演習2. '16. 06. 04

問1 (1)

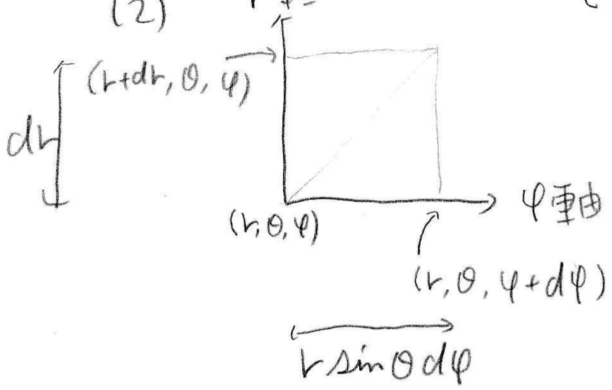


x−y平面の線を平行に上げる

別の角度から見ると



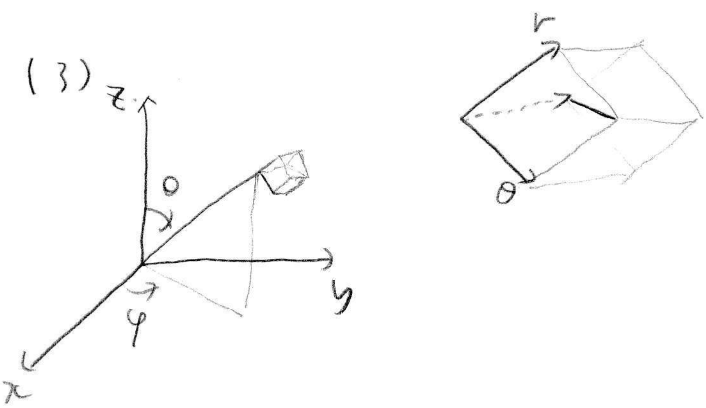
(2)



ピタゴラスの定理より

$$長さ = \sqrt{(dr)^2 + r^2\sin^2\theta\,(d\varphi)^2}$$

(3)



問2 (1) $ds = r\,d\theta\,dr$

(2) $\displaystyle\int_{\theta=0}^{\theta=2\pi} r\,dr\,d\theta = \Big[r\,dr\,\theta\Big]_{\theta=0}^{\theta=2\pi} = 2\pi r\,dr$

(3) $\displaystyle\int_{r=0}^{r=a} 2\pi r\,dr = 2\pi\left[\frac{r^2}{2}\right]_0^a = \pi a^2$